Jounrnal of Intelligence Community Research and Development
Category: Knowledge Discovery and Dissemination
Sponsor: IC Postdoc
Posted:26 October 2010
Type: Peer review

Cooperative Searching for Stochastic Targets

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Abstract

Spatial search problems abound in the real world, from locating hidden nuclear or chemical sources to finding skiers after an avalanche. We exemplify the formalism and solution for spatial searches involving two agents that may or may not choose to share information during a search. For certain classes of tasks, sharing information between multiple searchers makes cooperative searching advantageous. In some examples, agents are able to realize synergy by aggregating information and moving based on local judgments about maximal information gathering expectations. We also explore one- and two-dimensional simplified situations analytically and numerically to provide a framework for analyzing more complex problems. These general considerations provide a guide for designing optimal algorithms for real-world search problems.

1. Introduction

In the real world, there are many spatial search problems that involve multiple agents or searchers. Communication between such agents or between agents and a centralized command center may be sensitive, costly, or difficult for various reasons. In this work, we explore spatial search problems in this context and examine the classes of search problems for which communication and coordination between multiple agents will enable quantitative advantages over independent information gathering. These problems are very general and can be formalized and solved in terms of information theory. The solutions are essential to the development of quantitative decision support tools under uncertainty (e.g., in medicine [9]) and for automated multi-agent searches for stochastic information [1], such as those involving distributed sensor networks [19].

A search may be thought of as a series of steps by which an agent or agents reduce the uncertainty of the location of a target to zero. This location may be in physical space or in a "space of possibilities," i.e., a set of alternative scenarios. For example, if you need to find your keys in the morning before going to work and there are five rooms in your house, initially you know your keys must be in one of these rooms. After thoroughly searching one room and not finding the keys, your uncertainty is reduced; the keys must now be in one of the four remaining rooms, and so on. If you find the keys in the second room, then the uncertainty of their location is immediately zero because you know with complete certainty that they are in your hand.

Now imagine you have a friend help you search. The two of you would be searching twice as fast since, assuming you and your friend share information, a pair of searchers can eliminate rooms at a rate twice as fast as that of a single searcher. In more complex search problems, the target might emit some kind of complicated signal that makes it possible for multiple coordinated searchers to dramatically increase efficiency (imagine there are one thousand rooms to search but the keys are attached to some kind of radioactive homing beacon or can emit a sound). In these situations, understanding the nature of the clues and sharing information can be extraordinarily beneficial.

A search whereby agents move based *expressly* on information cues rather than following gradients is known as an infotaxis search. In a 2007 paper, Vergassola et al. explored infotaxis in the context of a moth following a pheromone trail through air to find a mate. The moth was performing a spatial search in turbulent air currents that carried the trail [18]. Compared to more conventional methods such as chemotaxis (following a chemical gradient [7,14]), infotaxis gives an advantage in situations when the signals from the target of the search are uncertain. This might be the case if signals from the target are stochastic, difficult to measure, or highly varying in time. In the case of the moth, the signal was sparse and widely dispersed by the turbulent air currents. When the moth followed the gradient of the strength of the trail directly – chemotaxis – it was forced to take a very circuitous route due to the turbulent dispersal of the trail. However, when the moth employed an infotaxis algorithm to move to positions where it might gain the most information about the source of the trail, its performance improved significantly. Since we will consider only search problems such as these, in the following we will refer to the target of an infotaxis search as the "source." The formalism of infotaxis balances the competing goals of exploiting the current information available and exploring to gain more information, a familiar compromise from other unsupervised learning methods, such as reinforcement learning [17]. While effective path planning algorithms may be based on the optimization of some objective function, these often rely on exploiting some features of the known environment [12] rather than a solid foundation based on information theory. Infotaxis provides a framework for studying search problems in general and is therefore broadly applicable.

In this article, we explore conditions in which cooperation between multiple infotaxis agents is advantageous. We focus on examples in which agents are able to realize synergetic cooperation by aggregating information and moving based on a local infotaxis algorithm. *Synergy*, and its opposite, *redundancy*, are information theoretic quantities that are defined in terms of relative probabilities of the stochastic variables involved [4]. In a recent paper, we showed that spatiotemporal correlations are necessary for synergy [11]. When synergy is exploited effectively it can lead to an exponential reduction in the search effort, in terms of time, energy, or number of steps [16,10,8]. Here we use a simple one-dimensional search example and a more realistic two-dimensional generalization to show how correlations lead to synergy. These simple examples provide a framework for analyzing more complex problems. Since, in general, the computational cost is greater for searchers to communicate and perform coordinated movements instead of moving based on independent decisions, we will describe situations in which coordination is worthwhile.

2. Information theory approach to stochastic search

Effective and robust search methods for locating stochastic sources balance the competing strategies of exploration and exploitation [17]. Given a current estimated probability distribution for the location of a source, a searcher might either exploit the data already collected by moving towards the location that maximizes this likelihood or sharpen the distribution (reduce uncertainty further) by moving to gather more diverse data. The infotaxis search balances these two strategies by optimizing the expected information gain over the possible next search moves. In the following, we review some basic concepts from information theory and formalize the infotaxis algorithm in terms of these quantities.

2.1 Information, synergy, and redundancy

To determine whether searchers can be effectively coordinated we define synergy and redundancy as information theoretical quantities [6] and use them as a measure of coordination. Synergy is found when measuring two or more variables *together* with respect to another (e.g., the source's signal) results in greater information gain than the sum of that from each variable *separately* [5,4]. In search problems, synergy is advantageous because then the coordination of two or more searchers is more efficient than the same searchers working independently. In this section we will introduce these concepts in general terms before applying them to a specific search problem.

Consider the stochastic variables $X_i, i = 1 \ldots n$. Each variable X_i can take on specific states, denoted by the corresponding lowercase letter x_i. For a single variable X_i the Shannon entropy (henceforth "entropy") is

$$S(X_i) = -\sum_{x_i} P(x_i) \log_2 P(x_i), \tag{1}$$

where $P(x_i)$ is the probability that the variable X_i takes on the value x_i [6]. The sum is over all of the possible states x_i; since $P(x_i)$ < 1 always, the entropy is always positive. The entropy is a measure of uncertainty about the state of X_i , therefore entropy can only decrease or remain unchanged as more variables are measured. The conditional entropy of a variable X_1 given a second variable X_2 is

$$S(X_1|X_2) = -\sum_{x_1,x_2} P(x_1,x_2) \log_2 \frac{P(x_1,x_2)}{P(x_2)} \leq S(X_1). \tag{2}$$

This expression contains a sum over the joint probability distribution of two variables. Since measuring a second variable can only decrease entropy (or leave it unchanged), the conditional entropy is bounded above by the entropy of the first variable. The mutual information between two variables, which plays an important role in search strategy, is defined as the change in entropy when a variable is measured:

$$I(X_1, X_2) = S(X_1) - S(X_1|X_2) \geq 0. \tag{3}$$

This is also the difference between the entropy of one variable and its entropy conditioned on the measurement of a second variable. Mutual information is always positive. These definitions can be directly extended to multiple variables. Just as entropy may be conditioned on an additional measurement, mutual information may be conditioned on the knowledge of other variables. These quantities may be used to generate new information theoretic constructs that we will use in specific search problems. For three variables [15] the quantity

$$R(X_1, X_2, X_3) \equiv I(X_1, X_2) - I(\{X_1, X_2\}|X_3) \tag{4}$$

measures the degree of "overlap" in the information contained in variables X_1 and X_2 with respect to X_3. The sign of this quantity is meaningful. Namely, if $R(X_1, X_2, X_3) > 0$, there is overlap, and X_1 and X_2 are said to be redundant with respect to X_3. If $R(X_1, X_2, X_3) < 0$, more information is available when these variables are considered together than when considered separately. In this case X_1 and X_2 are said to be synergetic with respect to X_3. If $R(X_1, X_2, X_3) = 0$, X_1 and X_2 are independent.

2.2 Bayesian inference and spatial infotaxis

We now formulate the general spatial stochastic search problem for N searchers seeking to find a stochastic source located in a finite, D-dimensional space. This is a generalization of the single searcher formalism presented in [18]. At any time step, the searchers s_i, $i = 1, 2, \ldots, N$, are located at position r_i and observe some number of particles h_i from the source. The searchers do not get information the trajectories or speed of the particles; they only get information if a particle was observed or not. Therefore simple geometrical methods such as triangulation are not possible.

Consider a random variable R_0, which can assume a number of specific values, denoted by r_0. The values of r_0 refer to positions in space that may contain the stochastic source. Only one value of r_0 corresponds to the (yet unknown) location of the source s_0. The searchers compute and share a probability distribution $P^{(t)}(r_0)$ for the source location at each time index t. Initially, the probability for the source $P^{(0)}(r_0)$ is assumed to be to be uniform. After each measurement $\{h_i, r_i\}$, the searchers update their estimated probability distribution of source positions via Bayesian inference [2] and decide what move to make (possibly remaining at the same position). The goal of Bayesian inference is to improve an estimated probability distribution $P(X)$, where X is a random variable that can assume a set of values denoted by $\{x\}$. Assuming that Y is another random variable (that can assume a set of values denoted by $\{y\}$) and that X and Y are not independent (that is, $I(X,Y) \neq 0$), knowledge of the state of Y can be used to improve $P(X)$. After a measurement reveals $Y = y$, the probability of this measurement given the current estimated $P(X)$ is computed. The probability of this measurement is $P(Y = y \mid X)$. Bayesian inference makes it possible to assimilate this information into the current estimate of $P(X)$ via a Bayesian update step: $P(X) \equiv P(X \mid Y = y) = P(Y = y \mid X)P(X)/A$, where

A is a normalization factor. This step includes an explicit statement of equivalence because each new measurement is included implicitly in $P(X)$. Therefore the measurement $Y = y$ improves the estimate of $P(X)$. The searchers will use this Bayesian inference framework to improve their estimate of the probability distribution of source locations $P^{(0)}(r_0)$ after each measurement $\{h_i, r_i\}$.

To decide where to move next, the searchers follow an infotaxis algorithm for multiple searchers. First the conditional probability

$$P^{(t+1)}(r_0|\{h_i, r_i\}) \equiv \frac{1}{A} P^{(t)}(r_0) P(\{h_i, r_i\}|r_0), \tag{5}$$

is calculated, where A is a normalization over all possible source locations r_0 as required by Bayesian inference. This is then assimilated via Bayesian update,

$$P^{(t+1)}(r_0) \equiv P^{(t+1)}(r_0|\{h_i, r_i\}). \tag{6}$$

If the searchers do not find the source at their present locations they choose the next local move using an infotaxis step to maximize the expected information gain. The expected information gain is computed in the following way; the entropy of the distribution $P^{(t)}(r_0)$ at time t is defined, using Eq. (1), to be

$$S^{(t)}(R_0) \equiv -\sum_{r_0} P^{(t)}(r_0) \log_2 P^{(t)}(r_0). \tag{7}$$

For a specific measurement $\{h_i, r_i\}$ the entropy *before* the Bayesian update is

$$S^{(t)}_{\{h_i, r_i\}}(R_0) \equiv -\sum_{r_0} P^{(t)}(r_0|\{h_i, r_i\}) \log_2 P^{(t)}(r_0|\{h_i, r_i\}). \tag{8}$$

We define the difference between the entropy at time t and the entropy at time $t+1$ after a measurement $\{h_i, r_i\}$ to be

$$\Delta S^{(t+1)}_{\{h_i, r_i\}} \equiv S^{(t+1)}_{\{h_i, r_i\}}(R_0) - S^{(t)}(R_0). \tag{9}$$

For a uniform prior, $P^{(0)}(r_0) = 1/M$ for M possible locations of the source in the discrete space, the entropy is maximum, $S^{(0)}(R_0) = \log_2 M$. For each possible joint move $\{r_i\}$, the change in expected entropy ΔS is computed and the move with the minimum (most negative) ΔS is executed.

The expected information gain is found by computing the entropy change for all of the possible joint searcher moves

$$\overline{\Delta S} = -\left[\sum_i P^{(t)}(R_0 = r_i)\right] S^{(t)}(R_0)$$
$$+\left[1 - \sum_i P^{(t)}(R_0 = r_i)\right] \Delta S^{(t+1)}_{\{h_i, r_i\}}$$
$$\times \sum_{h_1, h_2} \left[\sum_{r_0} P^{(t)}(r_0) P^{(t+1)}(\{h_i, r_i\}|r_0)\right]. \quad (10)$$

The first term in Eq. (10) corresponds to one of the searchers finding the source in the next time step (the final entropy will be $S = 0$ so $\overline{\Delta S} = -S$). The second term considers the reduction in entropy for all possible measurements at the proposed location, weighted by the probability of each of those measurements. The probability of the searchers obtaining the measurement $\{h_i\}$ at the location $\{r_i\}$ is given by the trace of the probability $P^{(t+1)}(\{h_i, r_i\}|r_0)$ over all possible source locations.

At each step the searchers move jointly to increase the expected information gain as measured by the change in entropy of the probability distribution. Although this algorithm is general in the following we consider only the case of two searchers ($N = 2$) and both one- and two-dimensional spatial domains.

3. Searching for correlated signals in one dimension

Sources that emit uncorrelated signals provide no opportunity for coordination because the searchers are never synergetic [11]. We instead consider signals with spatial, temporal, or other correlations. The simplest nontrivial example is searching in a one-dimensional domain for a source that emits two particles simultaneously in opposite directions. Two searchers should be able to exploit the correlations in the signal; if both searchers simultaneously observe particles, they can immediately conclude that the source is located between them. Therefore we expect synergy to be possible for some spatial arrangements of the source and searchers.

First consider a finite one-dimensional domain with a source s_0 and two searchers s_1 and s_2 at the corresponding positions $\{r_0, r_1, r_2\} \in [0, 1]$. The source is assumed to emit two particles simultaneously and in opposite directions. That is, one particle is emitted to the left and one to the right of the source. The two searchers s_1 and s_2 are identical with a fixed cross section such that $0 < a < 1$ is the probability of a searcher capturing one of the particles emitted from the source. At each step in the search the number of particle “hits” measured by searchers s_1 and s_2 are denoted by $h_1 \in \{0, 1\}$ and $h_2 \in \{0, 1\}$, respectively.

To calculate $R(r_0, h_1, h_2)$, it is first necessary to compute the probabilities of h_i for each searcher given the position of the source r_0. We note that since there is no distance dependence in the capture probability a , it is sufficient to consider three separate cases depending on the relative positions, or ordering, of the source and the searchers, as shown in **Figure 1**. For example, if s_1 is to the left of the source and s_2 is to the right of the source, the order is $s_1 s_0 s_2$. Note this is equivalent to the case $s_2 s_0 s_1$ since the searchers are identical.

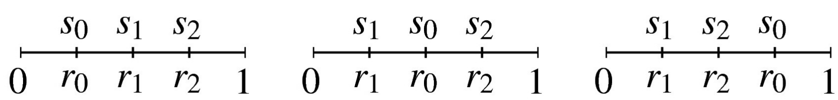

Figure 1: The three unique cases for relative positions of the source and two identical searchers on a one-dimensional domain. Since the probability of detection in this example does not depend on distance, we need only consider the spatial arrangement of the source and searchers. The cases are labeled $s_i s_j s_k$ according to the relative spatial ordering for the source s_0 and searchers s_1, s_2 . Since the searchers are identical, s_1 and s_2 are interchangeable and there are only three unique cases.

If a searcher observes a particle, it is assumed to be absorbed so that the other searcher will not be able to observe it. For example, if s_1 is between s_2 and the source (case $s_0 s_1 s_2$ or $s_2 s_1 s_0$), then the probability that s_2 observes a particle depends on whether s_1 observed it. If s_1 observed it, then the probability that s_2 observes it must be 0. If s_1 did not observe it, then the probability that s_2 will observe it is a

$$P(h_2 = 1 | h_1 = 1, r_0) = 0, \tag{11}$$

$$P(h_2 = 1 | h_1 = 0, r_0) = a. \tag{12}$$

We can use the probability relation

$$P(h_2, h_1 | r_0) = P(h_2 | h_1, r_0) P(h_1 | r_0) \tag{13}$$

to compute the two-searcher conditional probabilities

$$P(h_2 = 1, h_1 = 1 | r_0) = 0, \tag{14}$$

$$P(h_2 = 1, h_1 = 0 | r_0) = a(1 - a). \tag{15}$$

The other probability distributions are computed using similar reasoning. The results are summarized in **Table 1**.

Cases	h_1, h_2	$P(h_1\|r_0)$	$P(h_2\|r_0)$	$P(\{h_1, h_2\}\|r_0)$
$s_0s_1s_2$ $s_2s_1s_0$	$1,1$	a	$a(1-a)$	0
	$1,0$	a	$1-a(1-a)$	a
	$0,1$	$1-a$	$a(1-a)$	$a(1-a)$
	$0,0$	$1-a$	$1-a(1-a)$	$(1-a)^2$
$s_1s_0s_2$ $s_2s_0s_1$	$1,1$	a	a	a^2
	$1,0$	a	$1-a$	$a(1-a)$
	$0,1$	$1-a$	a	$a(1-a)$
	$0,0$	$1-a$	$1-a$	$(1-a)^2$
$s_0s_2s_1$ $s_1s_2s_0$	$1,1$	$a(1-a)$	a	0
	$1,0$	$a(1-a)$	$1-a$	$a(1-a)$
	$0,1$	$1-a(1-a)$	a	a
	$0,0$	$1-a(1-a)$	$1-a$	$(1-a)^2$

Table 1: Conditional probabilities for the six possible arrangements of the source s_0 and searchers s_1, s_2 shown in **Figure 1**. Since the searchers are identical, s_1 and s_2 are interchangeable and there are only three unique cases. In cases $s_0s_1s_2$ and $s_2s_1s_0$, searcher one is between the source and searcher two. A particle emitted by the source in the direction of the searchers will reach searcher one first. If searcher one detects the particle, searcher two will not be able to detect it. Searcher two will only have a chance of detecting the particle if the particle passes through searcher one undetected. Similarly, in cases $s_0s_2s_1$ and $s_1s_2s_0$, searcher two is between the source and searcher one. A particle emitted by the source in the direction of the searchers will reach searcher two first. If searcher two detects the particle, searcher one will not be able to detect it. Searcher one will only have a chance of detecting the particle if the particle passes through searcher two undetected. In cases $s_1s_0s_2$ and $s_2s_0s_1$, the source is between the searchers and the searchers do not interfere with each other. Furthermore, since the source emits

two particles simultaneously in opposite directions, in only these cases do both searchers have a chance of each detecting a particle.

Using **Table 1**, we can analytically compute the information theoretic quantities we need to determine synergy and redundancy. These are

$$R(h_1, h_2, r_0) = I(h_1, h_2) - I(h_1, h_2|r_0), \tag{16}$$

$$I(h_1, h_2) \equiv \sum_{h_1, h_2} P(h_1, h_2) \log_2 \frac{P(h_1, h_2)}{P(h_1)P(h_2)}, \tag{17}$$

and

$$I(h_1, h_2|r_0) \equiv \sum_{h_1, h_2} \int_0^1 dr_0 P(h_1, h_2, r_0) \times \log_2 \frac{P(h_1, h_2|r_0)}{P(h_1|r_0)P(h_2|r_0)}; \tag{18}$$

where the probability distributions are calculated as follows:

$$P(h_1, h_2, r_0) = P(h_1, h_2|r_0)P(r_0), \tag{19}$$

$$P(h_1, h_2) = \int_0^1 dr_0 P(h_1, h_2|r_0)P(r_0), \tag{20}$$

$$P(h_1) = \int_0^1 dr_0 P(h_1|r_0)P(r_0), \tag{21}$$

$$P(h_2) = \int_0^1 dr_0 P(h_2|r_0)P(r_0). \tag{22}$$

Initially we consider a uniform probability distribution (prior) of source locations: $P(r_0) = 1$.

For small capture probability, a, we can expand Eq. (16) as a Taylor series in a to get an analytical solution for the critical values a_c where $R|_{a=a_c} = 0$. For $r_1 > r_2$, the critical values are given by

$$a_c = \sqrt{\frac{3(r_2 - r_1)\log(r_1 - r_2)}{r_1^3 - r_2^3 - 3r_1^2 + 2r_1 + r_2}}. \tag{23}$$

For $r_1 < r_2$, a_c is given by Eq. (23) under the transformations $r_1 \to r_2$ and $r_2 \to r_1$. These relations give the approximate boundary between the regions of synergy and redundancy. In the limit $r_1 \to r_2$, $R \to 0$. For these formulas we used a uniform source distribution $P(r_0) = 1$ but it is possible to repeat these calculations with a different $P(r_0)$.

For larger values of a when the expansion is no longer valid, the condition $R = 0$ [as in Eq. (16)] can be solved numerically. **Figure 2** shows $R(h_1, h_2, r_0)$ as a function of the capture probability a and searcher s_1 location r_1 with searcher s_2 fixed at $r_2 = 2/3$. This figure also shows $R(h_1, h_2, r_0)$ for a Gaussian probability of the source location $P(r_0) = B\exp(-(r_0 - 1/3)^2)$, where B is a normalization factor. In both cases, $R > 0$ (indicating redundancy) when the searchers are close together and $R < 0$ (indicating synergy) when they are farther apart. For the Gaussian distribution source location, synergy is strongest when s_1 is close to the source because the mutual information between r_0 and h_1 is peaked there as well.

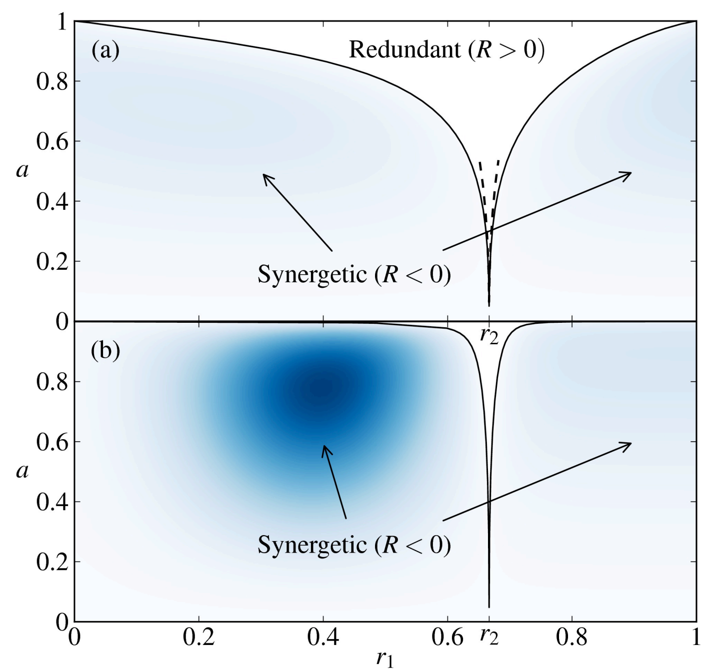

Figure 2: Synergetic and redundant regions for two searchers in the one-dimensional correlated search problem. The value of $R(h_1, h_2, r_0)$ is shown for different locations r_1 of s_1 with s_2 held fixed at $r_2 = 2/3$ and the capture probability varying from $a = 0$ (no capture) to $a = 1$ (complete capture). Darker regions represent higher synergy (larger negative values of R) and the white areas represent redundancy. The critical contour line $R = 0$ separates the synergetic and redundant regions (a). When the source is equally likely to be anywhere, $P(r_0) = 1$, there is only weak synergy (b). Then, the probability of the source is a Gaussian distribution centered at $1/3$, $P(r_0) \propto \exp(-(r_0 - 1/3)^2)$, the synergy is highest near the source at $1/3$ since the mutual

information is highest there as well. The dashed line in (a) shows the critical values a_c computed by the approximation in Eq. (23); this approximation is good for $a_c \lesssim 0.3$.

Figure 2(a) shows how the capture probability a influences the possible strategies of the searchers. If a is close to 1, then the searchers only realize synergy if they are far apart, which maximizes the chance of the source being between them. But if a is is close to 0, then nearly any arrangement of searchers is synergetic but only weakly. **Figure 2(b)** demonstrates the significance of the prior $P(r_0)$ in the calculation of $R(h_1, h_2, r_0)$. The variance of $P(r_0)$ is equal to 1, the size of the domain, making the Gaussian distribution quite broad. Nonetheless, the fact that the prior is weakly peaked at some point in the domain dramatically reduces the area of the redundant ($R > 0$) region and allows for much greater synergy between the searchers. Although search problems in the real world are rarely one-dimensional, this example illustrates the basic calculations for determining synergy.

4. Searching for correlated signals in two dimensions

The one-dimensional example provides a tractable starting point for generalization to two-dimensional problems. The simplest generalization is the case of two mobile searchers and one stationary source in a two-dimensional finite domain. Imagine a source in which chemical or nuclear reactions are occurring and the products of the reactions leave the source with relative angular correlations. Our two-dimensional idealized problem consists of a source that emits 2 particles simultaneously at each time step in opposite directions along some emission axis. At each time step, a new emission axis angle is chosen uniformly at random from $[0, 2\pi]$. The particles move along straight-line trajectories, but the searchers are not able to measure the velocities of any detected particles, so geometric methods such as triangulation are not possible.

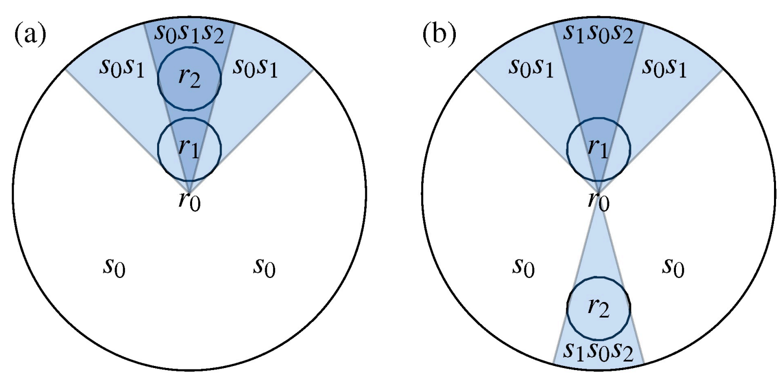

Figure 3: Diagram for the two-dimensional example. The source position r_0 is fixed in the center and the searchers are disks of radius d. At each time step, the source emits two particles in in opposite directions with a random angle. Each possible emission axis passing through the source corresponds to one of six cases for source and searcher configurations as given in **Tables 1** and **2**.

In the real world, for example, searchers could be autonomous mobile sensors capable of detecting radiation or other reaction products from the above example. Most modern-day autonomous robots do not move much faster than a walking pace [13], largely due to the difficulties of navigating uncertain terrain safely. Thus movement is relatively costly (backtracking will take a significant amount of time) and the searchers should make decisions to refine their trajectories often, using any new available information. For this problem, this means that the searchers can only make small discrete movements between measurements.

To represent simplified mobile autonomous robots, we cast the searchers as identical disks of radius d that move on a regular Cartesian grid. Unlike the one-dimensional case, these searchers have spatial extent; there are two parts to the calculation of particle detection. First, if a particle travels along a straight line trajectory that passes through a searcher, the capture probability is $0 < a < 1$. As in the one-dimensional example, if a particle is observed by a searcher, it is absorbed so it cannot be observed by the other searcher. Second, we must consider the probability that the particle's trajectory will pass through the searcher. This is a function of the searcher radius d and the distance to the source. The variables r_0, r_1, and r_2 for the positions of the source and searchers each have two components, e.g.$(r_{0,x}, r_{0,y})$, since they represent positions on a two-dimensional grid.

As in the one-dimensional example, there are different cases for the probabilities which depend on the relative position of the two searchers to the source. For all possible straight line emission axes passing through the source, some lines may pass through no searchers (case s_0), only searcher s_1 (case s_0s_1), or only searcher s_2 (case s_0s_2). If a line passes through both searchers, the source is between the searchers (case $s_1s_0s_2$ as in the one-dimensional example), or one of the other searchers is in front of the other (cases $s_0s_1s_2$ and $s_0s_2s_1$ as in the one-dimensional example). These cases are illustrated in **Figure 3**. For any source location, there will be a range of angles $\Delta\theta_c$ for each case c. The probabilities for the cases that do not appear in the one-dimensional example (see **Table 1**) are detailed in **Table 2**. Quantities such as $P(h_1, h_2|r_0)$ are a superposition of the values for the different cases, weighted by the proportion of angles corresponding to each case

$$P(h_1, h_2|r_0) = \sum_{c\in\{\text{cases}\}} \frac{\Delta\theta_c}{2\pi} P_c(h_1, h_2|r_0), \tag{24}$$

and similarly for other quantities such as $P(h_1|r_0)$, etc. In principle, while it is possible to find the $\Delta\theta_c$ analytically using geometry, in practice, it is much more efficient to do this numerically.

Case	h_1, h_2	$P(h_1\|r_0)$	$P(h_2\|r_0)$	$P(\{h_1, h_2\}\|r_0)$
s_0	$1,1$	0	0	0
	$1,0$	0	1	0
	$0,1$	1	0	0
	$0,0$	1	1	1
$s_0 s_1$	$1,1$	a	0	0
	$1,0$	a	1	a
	$0,1$	$1-a$	0	0
	$0,0$	$1-a$	1	$1-a$
$s_0 s_2$	$1,1$	0	a	0
	$1,0$	0	$1-a$	0
	$0,1$	1	a	a
	$0,0$	1	$1-a$	$1-a$

Table 2: Conditional probabilities for cases unique to the two dimensional example in **Figure 3**. The cases $s_1 s_0 s_2$, $s_0 s_1 s_2$, and $s_0 s_2 s_1$ (not shown), are identical to those of the one dimensional problem given in **Table 1**. As shown in **Figure 3**, the case s_0 corresponds to the range of angles for which it is impossible for either searcher to detect a particle. Case $s_0 s_1$ corresponds to the range of angles for which only searcher one has a chance of detecting a particle, and case $s_0 s_2$ corresponds to the range of angles for which only searcher two has a chance of detecting a particle.

Note that the different cases in the one-dimensional example were taken into account implicitly during integration as in Eq. (22). In the two-dimensional example, they are taken into account when the effective probability distributions are computed as a superposition of the probabilities of the individual cases. This is because there is more than one case for each source location relative to the searchers.

We illustrate the two-dimensional searcher problem with a setup of searcher and source locations that illustrate the synergetic and redundant positions. We use the Gaussian prior for the initial guess

$$P(r_0) = A \exp - \frac{||r_0 - s||^2}{\sigma^2}, \tag{25}$$

where A is the normalization and σ determines the overall shape of the distribution. The vector $s = (2, 2)$ is the most probable position of the source. The quantity $R(h_1, h_2, r_0)$ [see Eq. (4)] determines whether the searchers are positioned synergetically relative to the source. When the searchers are performing an infotaxis search, realizing synergetic relative positions will in principle lead to the fastest reduction in uncertainty.

In **Figure 4** we plot R as a function of the position of one searcher r_1 with the position of the second searcher r_2 held fixed. We find for this example that only synergy ($R < 0$) is possible. The light areas in **Figure 4** correspond to weak synergy and the dark areas to stronger synergy. As in the one-dimensional example, synergy is strongest for large a and when searcher s_1 is near the source and not behind searcher s_2. The small cross section [$a = 0.25$ in Figure 4(a)] allows only for weak synergy, whereas a larger cross section [$a = 0.75$ in Figure 4(b)] gives much stronger synergy for certain relative positions. The strongest synergy comes from a large cross section paired with optimal positioning. The maximum synergy is realized when searcher s_1 is close to the peak of P and especially when the peak of P is between the searchers. Note that unfavorable positions (such as $r_1 = (5, 2)$, behind searcher s_2) provide minimal synergy regardless of the value of a.

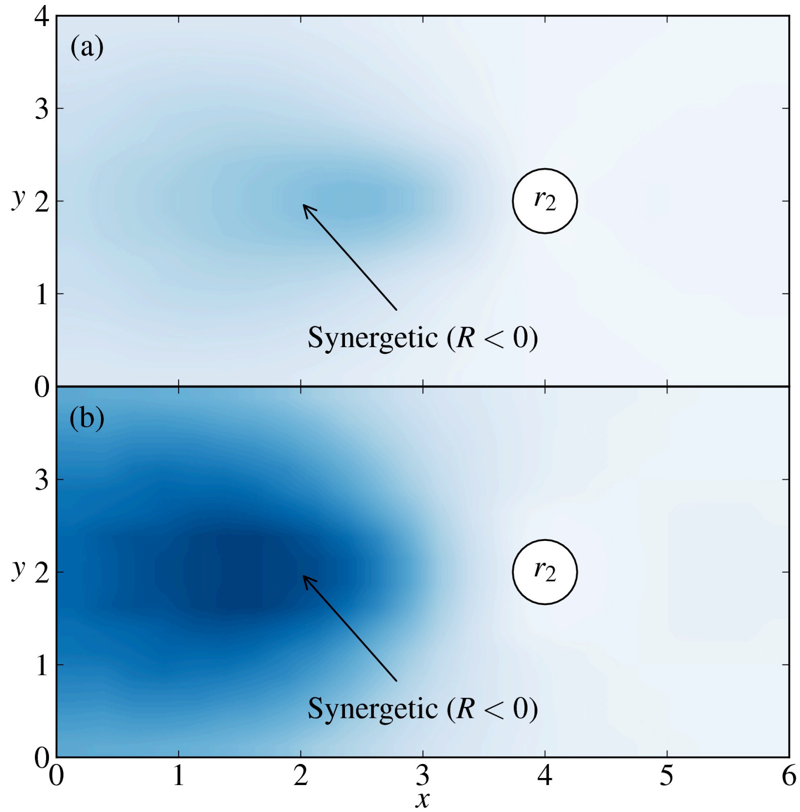

Figure 4: Synergy for two searchers for two-dimensional correlated signals. The value of $R(h_1, h_2, r_0)$ is shown as a function of the position r_1 of searcher s_1 for two different values of the capture probability a : (a) $a = 0.25$; (b) $a = 0.75$. The position of searcher s_2 is fixed $r_2 = (2, 4)$. The most probable source location (the peak of the Gaussian distribution for the initial source location) is at $r_0 = (2, 2)$. Darker blue corresponds to stronger synergy ($R < 0$). Synergy is strongest for large a and when searcher s_1 is near the source and not behind searcher s_2 relative to the source.

For real-world problems, larger cross sections yield more information and therefore stronger synergy is possible. However for some applications, the cross section of a sensor on a searcher may be limited by practical considerations such as weight or power consumption. This example shows that synergy is still possible for small a . Furthermore, this example emphasizes the importance of the probability estimate of source locations $P(r_0)$. The infotaxis algorithm is designed such that the searchers will explore, gathering new information, if their arrangement is not sufficiently synergetic to warrant exploitation. This allows the searchers to succeed even with no starting information, but they may not realize strong synergy until the estimate of $P(r_0)$ is sufficiently refined.

5. Conclusion

In this work we studied search algorithms for autonomous agents looking for the spatial location of a stochastic source. In spatial search problems, since the exploitation of synergy requires spatial or temporal correlations, we considered problems in which a source emits two particles simultaneously in opposite directions. This is a simplification of physical problems in which there is a reaction and the products travel in directions that have angular correlations. We showed that both synergy and redundancy are possible for one-dimensional search problems but not for two-dimensional searches, where only synergy is possible. Since even unfavorable arrangements of searchers are synergetic, in two-dimensional search problems like these coordination is *always* advantageous.

Simple examples such as these that can be studied analytically provide insight into real-world problems. In real-world problems, there will necessarily be additional considerations. It may not always be possible to write a closed-form equation for the nature of the correlations in the signal from the source. It may be necessary to directly measure any correlations and use this to estimate capture probabilities. Furthermore, various probabilities may not be stationary in time or the signals from the source may get progressively weaker. For example, signals from a radio transmitter may decrease in strength over time as its batteries are slowly exhausted. An additional consideration is that in the real-world, communication between agents may only be possible at certain times and may not be instantaneous as in our simple examples. These general considerations are crucial for the exploitation of multi-agent infotaxis in terms of the design of optimal collective algorithms in particular applications. The next steps for making this approach applicable to a broader class of problems, including those not limited to spatial searches[3], are to generalize the results to more than two searchers and to explore how synergy may be best leveraged to give increases in search speed and efficiency.

Acknowlegement

This work was supported in part by a DCI IC Postdoctoral Research Fellowship and the Department of Energy at Los Alamos National Laboratory under contract DE-AC52-06NA25396 through the Laboratory Directed Research and Development Program.

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